

Survey of Quantum-Inspired Evolutionary Algorithms

Abstract. This paper presents a concise survey of a new class of metaheuristics, drawing their inspiration from both: biological evolution and unitary evolution of quantum systems. In the first part of the paper, general concepts behind quantum-inspired evolutionary algorithms have been presented. In the second part, a state of the art of this field has been discussed and a literature review has been conducted.

Streszczenie. Niniejszy artykuł przeglądowy dotyczy nowej klasy metaheurystyk przeszukiwania, czerpiących inspirację zarówno z ewolucji biologicznej jak i z ewolucji unitarnej układów kwantowych. W pierwszej części artykułu zostały przedstawione podstawowe idee leżące u podstaw kwantowo inspirowanych algorytmów ewolucyjnych. Druga część artykułu ma charakter przeglądu literatury w tym obszarze oraz przedstawia aktualny stan wiedzy.

Keywords: evolutionary computing, quantum computing, quantum-inspired evolutionary algorithms, genetic algorithms

Słowa kluczowe: obliczenia ewolucyjne, informatyka kwantowa, kwantowo inspirowane algorytmy ewolucyjne, algorytmy genetyczne

1 Introduction

This paper concerns a new class of metaheuristics, drawing their inspiration from two fields: *evolutionary* and *quantum computing*. Quantum computing[44] is a branch of computer science concerning applications of unique quantum mechanical effects to solving computational problems. The first theoretical studies on quantum computing has been started in 1980s[4]. In theory, by application of unique quantum mechanical phenomena it is possible to solve selected computational problems extremely efficiently[17, 51]. However, there are still several serious difficulties in building a functional and scalable quantum computer. It is possible to simulate quantum computation on a classical computer, yet it is highly inefficient and computationally exhaustive. Nevertheless, quantum computing remains a valuable source of inspiration for contemporary computer science. In quantum evolutionary computing, the following interplay between the subareas is possible[70]:

1. Evolutionary synthesis of quantum algorithms[16, 49]
2. Quantum Evolutionary Algorithms (QEAs) [39, 52, 57]
3. Quantum-Inspired Evolutionary Algorithms (QIEAs)

This paper concerns the quantum-inspired evolutionary algorithms, and this approach does not require a functional quantum computer. The goal of this paper was to present fundamentals of the algorithms and a brief review of the most significant research efforts in this field from the past decade.

This paper is arranged as follows. In section 2, quantum-inspired evolutionary algorithms have been briefly summarized. In section 3, a literature review has been conducted, methods of the survey have been described and some statistical summaries have been presented. In section 4, conclusions have been drawn and some forecasts of possible future research paths have been given.

2 Quantum-Inspired Evolutionary Algorithms

The *quantum-inspired evolutionary algorithms* are located in the intersection of two subareas of computer science, quantum computing and evolutionary computing, which has been presented in figure 1.

Contrary to "true" quantum algorithms (e.g. Grover's search algorithm[17] or Shor's factorization algorithm[51]), the considered algorithms do not require a functional quantum computer for their efficient implementation. Instead, the algorithms exploit additional level of randomness inspired by concepts and principles drawn from quantum mechanical systems, such as *qubits*, *interference* or *superposition of states*.

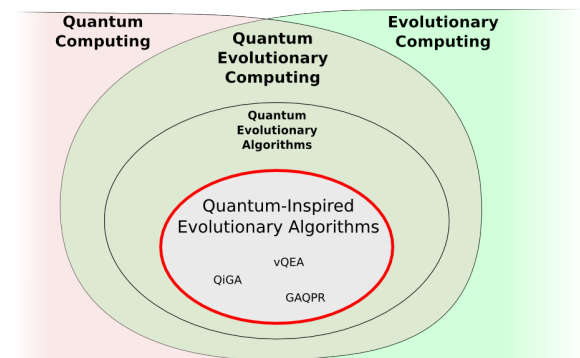


Fig. 1. Quantum and evolutionary computing subfields

The first proposal of evolutionary algorithm based on the concepts and principles of quantum computing was presented in [42] and this area is still intensively studied nowadays. Other early examples of quantum-inspired genetic algorithms which employ binary quantum representation based on qubits are due to Han and Kim[21, 22, 23]. In the past decade, several other variants[9, 8, 69, 71, 13] of quantum evolutionary algorithms have been also proposed.

In figure 2, the pseudo-code of general quantum-inspired evolutionary algorithm has been presented. In the figure, $Q(t)$ denotes the t -th generation of a quantum population.

```

procedure QIEA
begin
   $t \leftarrow 0$ 
  initialize  $Q(0)$ 
  ...
  while not termination-criterion do
     $t \leftarrow t + 1$ 
    ...
    evaluate  $Q(t)$ 
    perform genetic operators on  $Q(t)$ 
    ...
  end while
end

```

Fig. 2. Quantum-Inspired Evolutionary Algorithm pseudo-code

It is easy to see that the pseudo-code corresponds directly to the general classical evolutionary algorithm scheme. However, the main stages of quantum-inspired evolutionary algorithm are modelled upon concepts and principles of quantum computing.

2.1 Quantum Elements in Evolutionary Algorithms

The significant feature of the new algorithms is *representation of solutions*. Contrary to traditional evolutionary algorithms, a *quantum population* consists of probability distributions of sampling the search space, instead of exact points in the space. However, in the special case, a *quantum individual* can also indicate an exact element of the search space. This generalization brings a "new dimension" into evolutionary algorithm. Moreover, quantum-inspired elements can be also introduced in the main stages of evolutionary algorithm. The stages have been emphasized in figure 3.

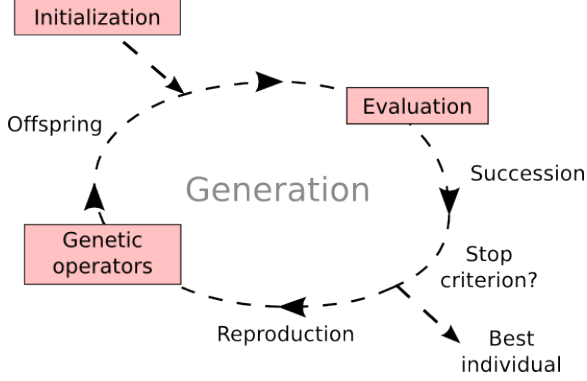


Fig. 3. General scheme of Quantum-Inspired Evolutionary Algorithm

Basic possibilities of introducing quantum-inspired elements into the main stages of evolutionary algorithms are as follows:

1. *Initialization* – Initial quantum population $Q(0)$ does not have to specify exact elements of the search space. Instead, it can be initialized by some probability distributions of sampling the space. Uniform distributions or other families of distributions with random parameters can be used.
2. *Evaluation of individuals' fitness* – quantum individuals are chosen for reproduction according to their fitness. Various stochastic and deterministic measures of quality for quantum genotypes are possible at this stage of the algorithm.
3. *Genetic Operators* – genetic operators in quantum-inspired evolutionary algorithms are adapted to the new representation. Two possible groups of the new genetic operators can be identified: (1) generalization of classical genetic operators to the new quantum representation and (2) a new class of operators, modelling directly quantum mechanical phenomena (e.g. rotations of quantum system state vectors).

2.2 Quantum-Inspired Genetic Algorithm

The Quantum-Inspired Genetic Algorithms (QIGA) has been proposed in [21] and the algorithm has been briefly presented in this section. In QIGA algorithm, a novel representation of solutions, binary quantum coding, is employed. Instead of bits, quantum genes are modelled upon the concept of *qubits*, which brings an additional element of randomness into the algorithm. Qubit is a basic unit of quantum information. It is a normalised vector in a two dimensional vector space spanned by the base vectors $|0\rangle$ and $|1\rangle$, as given in equation:

$$(1) \quad |\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where: $\alpha, \beta \in \mathbb{C}$, $|0\rangle = [1 \ 0]^T$, $|1\rangle = [0 \ 1]^T$ and $|\alpha|^2 + |\beta|^2 = 1$.

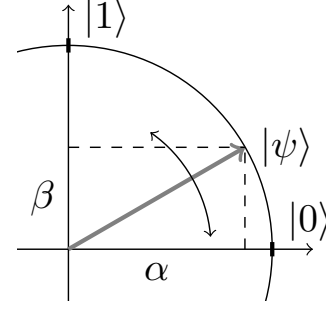


Fig. 4. Illustration of binary quantum gene state

With some simplification (imaginary part omitted), a state of binary quantum gene $|\Psi\rangle$ can be depicted as a unit vector which has been presented in figure 4. Along with increase of the angle between the vector and the horizontal axis, the probability of observing value 1 grows, while the more horizontal direction of the vector, the higher probability of observing value 0. QIGA algorithm uses *binary quantum chromosomes* for representation of solutions, encoded as:

$$(2) \quad q = \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_m \\ \beta_1 & \beta_2 & \dots & \beta_m \end{bmatrix}$$

where each column corresponds to binary quantum gene $|\Psi_1\rangle, \dots, |\Psi_m\rangle$. Hence, a state of the whole quantum population $Q = \{q_1, q_2, \dots, q_N\}$ can be simply illustrated by a matrix of vectors, which has been presented in figure 5. Each row in the figure corresponds to binary quantum chromosome, as in (2).

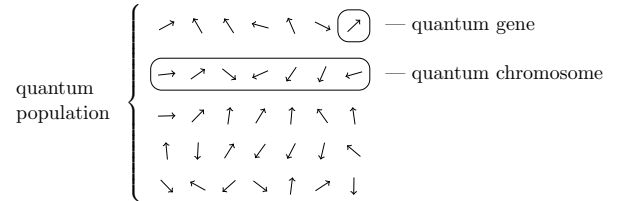


Fig. 5. Illustration of binary quantum population. Each arrow represent a state of a quantum gene

The full pseudo-code of QIGA algorithm has been presented in figure 6.

```

procedure QIGA
begin
   $t \leftarrow 0$ 
  initialize  $Q(0)$ 
  make  $P(0)$  by observing  $Q(0)$ 
  evaluate  $P(0)$ 
  store the best solution among  $P(0)$ 
  while not termination-criterion do
     $t \leftarrow t + 1$ 
    make  $P(t)$  by observing  $Q(t - 1)$  states
    evaluate  $P(t)$ 
    update  $Q(t)$  using quantum gates  $U(t)$ 
    store the best solution among  $P(t)$ 
  end while
end
  
```

Fig. 6. Quantum-inspired Genetic Algorithm [21]

In the beginning of the algorithm, the genes of all individuals in the quantum population $Q(0)$ are initialized with linear superposition $(\frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|1\rangle)$, which results in sampling the whole search space with equal probability. During a phenotype creation, states of all genes in quantum chromosomes are *observed*, i.e. the search space is sampled with respect

to the probability distribution encoded in the quantum chromosomes. The evaluation of individual's fitness is based on the observed classical population $P(t)$. The genetic operators applied in the algorithm are based on *quantum rotation gates*, which rotate state vectors in the quantum gene state space. For more detailed description of the algorithm, the reader is referred to [21].

In recent years, quantum-inspired genetic algorithms have been receiving growing attention. Numerous extensions of QIGA algorithm have been proposed, including Genetic Algorithm based on Quantum Probability Representation(GAQPR)[5], versatile quantum evolutionary algorithm(vQEA)[11] and novel quantum-inspired evolutionary algorithm(NQEA)[38].

3 Literature Review

In this section, a collection of over 160 bibliography items regarding quantum-inspired evolutionary computing has been concisely summarized. The collection covers a vast majority of research in this field. It has been assembled by the author of the present paper since autumn 2008. The bibliography section of this paper presents an excerpt from the whole collection. Upon the reader's request, the author can provide the whole bibliography database in BibTeX format.

3.1 Methods

The material of the survey has been extracted from several sources of technical engineering literature including online services provided by: Elsevier, IEEE, Springer, ScienceDirect, EBRARY and arXiv. The survey concerns published contributions including books, reports, journal or magazine publications and conference proceedings. In addition, PhD theses have been also included, regardless published anywhere or not. The Google Alerts service has been used to follow newest contributions constantly.

3.2 Statistical Summaries

In this subsection, some general statistical summaries of quantum-inspired evolutionary algorithms literature have been given.

In table 1, annual distribution of publications has been presented. The most popular authors and distribution of publication type have been presented in table 2 and table 3, respectively. Each item in the collection has been given at least one describing tag (e.g. application, survey, theoretical etc.). The tags containing most items have been presented in table 4.

Table 1. Annual distribution of publications

Year	Items
1996	1
2000	2
2001	3
2002	2
2003	8
2004	9
2005	6
2006	9
2007	14
2008	34
2009	37

Table 2. The most popular authors

Author	Items
K.H. Han	15
J.H. Kim	13
G. Zhang	7
L. Wang	6
Y. Li	5

Table 3. Distribution of publication type

Type	Items
journal article	79
conference proceedings	66
PhD theses	2
books	1

Table 4. The most popular tags

Tag	Items
Application	79
Genetic operators	42
Representation	23
Knapsack	20
Real-coded	14
...	...
Theoretical analysis	5
Survey	2

3.3 Discussion

The first proposal of the algorithm drawing inspiration from both biological evolution and unitary evolution of quantum systems has been presented by Narayanan and Moore[42] in 1996. Further early studies and proposals in this field are due to Han and Kim[21, 26, 23, 28, 27].

In table 4, it is easy to see that most papers in the past decade reported successful applications of quantum-inspired evolutionary algorithm in different problems. In 42 papers, a variety of extended, generalized or modified genetic operators (e.g. [69, 67, 30, 8, 56]) for quantum individuals have been introduced. In 23 papers, various modifications of quantum individuals' representation (e.g. [73, 36, 12, 47, 31, 37, 71]) were discussed. Most papers concerns binary quantum coding (qubits instead of bits), and only a tiny fraction of all papers [7, 8, 29, 2, 47, 32] concerns real-coded algorithms. Very few papers [22, 28, 24, 25, 35] are focused mainly on in-depth theoretical analysis. Only two comprehensive surveys[16, 70] on this field have been conducted. Two PhD theses[27, 50] and one book[43] have been devoted to quantum-inspired evolutionary algorithms entirely.

Many researchers have reported efficacy of the quantum-inspired evolutionary algorithms in several dozen combinatorial and numerical optimization problems. The last decade has witness successful applications of the algorithms in variety of different areas, including image processing [53, 54, 15, 68, 34], network design problems[63, 64, 65, 62, 48], flow shop scheduling problems[61, 18, 19, 72, 45], power system optimization[58, 59, 41], thermal unit commitment[33], discovering structures in time series[5] and others[20, 55, 60, 66]. Quantum evolutionary algorithms with real numbers representation have been also successfully applied in various fields: engineering optimization problems[2, 3], option pricing modelling[12, 14], power system optimization[1], financial data analysis[10, 13, 14, 12], training fuzzy neural networks[71] and ceramic grinding optimization[40]. Consequently, it has been demonstrated

that quantum evolutionary algorithms are capable to outperform classical metaheuristics for a wide range of problems.

Though several successful attempts (e.g. [28, 25, 35]) have been made for some specific algorithms, no strong theoretical foundations of general quantum evolutionary algorithms exist or the models are highly oversimplified. Also, no systematic approach to tuning parameters of the algorithms has been taken yet. Therefore, no general rules for setting parameters of quantum evolutionary algorithms are currently identified, and the researcher's experience in setting the parameters is usually required.

4 Conclusions and Future Research Paths

In this paper, quantum-inspired evolutionary algorithms have been briefly presented. Special attention has been stressed to quantum-inspired genetic algorithms and the first proposal of such algorithm has been shortly recalled. The literature review has been conducted and its statistical summaries have been provided.

The additional elements of randomness bring a new dimension into evolutionary algorithms. Further studies on how to use the new possibilities efficiently can be expected in near future. The new dimension of randomness in quantum-inspired evolutionary algorithms allows the algorithms to be finely adjusted to match various optimization problems. Moreover, the additional elements of randomness can bring the algorithms closer to vague nature of many real-world problems. Thus, it can be expected that quantum-inspired evolutionary algorithms are particularly useful in solving inherently uncertain, imprecise problems, i.e. evolving rough sets or fuzzy rules.

Also, theoretical studies of the algorithm still need better attention. Examples of such possible theoretical research paths include: generalization of Holland's schema theorem to quantum-inspired genetic algorithms or an analysis in dynamic systems perspective. Machine learning approach[6] and modern metaheuristics tuning techniques (like [46]) are also interesting future research paths.

Ultimately, possible implementation of the quantum evolutionary algorithms on quantum computers is the biggest challenge for the future. Some early promising yet theoretical results of this approach has been already achieved [39, 52, 57].

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