

# SURVEY OF QUANTUM-INSPIRED EVOLUTIONARY ALGORITHMS

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Computer Engineering Department  
Technical University of Lodz

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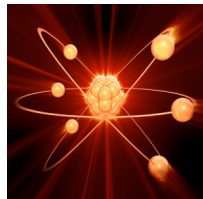


# OUTLINE

- 1 Background Information and Scope of Research
- 2 Quantum + Evolutionary Computing
- 3 Quantum Elements in Evolutionary Algorithms
- 4 Current problems
- 5 My Contributions
- 6 Preliminary Results and Selected Applications

# QUANTUM COMPUTING

**Quantum Computing** – branch of theoretical computer science dealing with application of **quantum mechanical** effects to solving computational problems.



# QUANTUM + EVOLUTIONARY COMPUTING

Possible interplay between quantum and evolutionary computing:

- 1 Evolutionary-Designed Quantum Algorithms  
Automatic synthesis of quantum algorithms
- 2 Quantum-Inspired Evolutionary Algorithms (QIEAs)  
Modification of existing EA algorithms by application of quantum-*inspired* concepts and principles
- 3 „True” Quantum Evolutionary Algorithms  
Ultimately, implementation of the algorithms in a “true” quantum level hardware is the biggest challenge for the future

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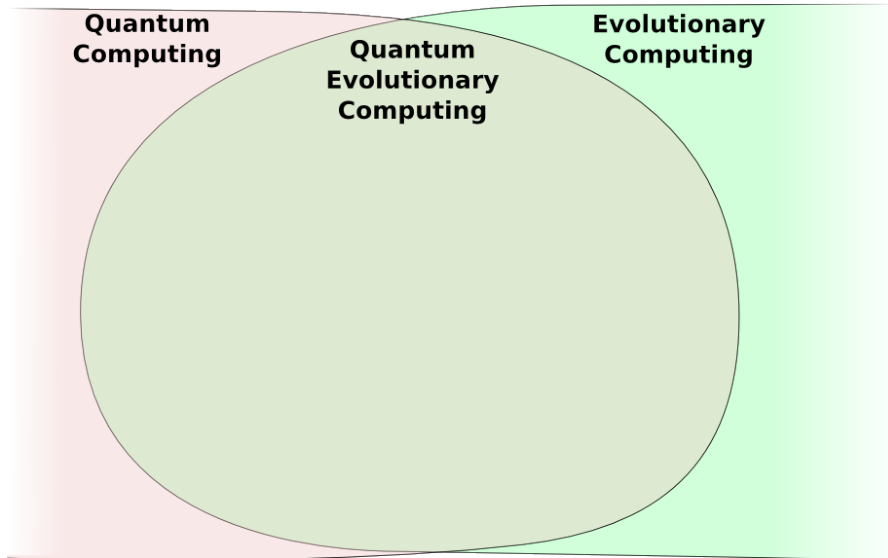
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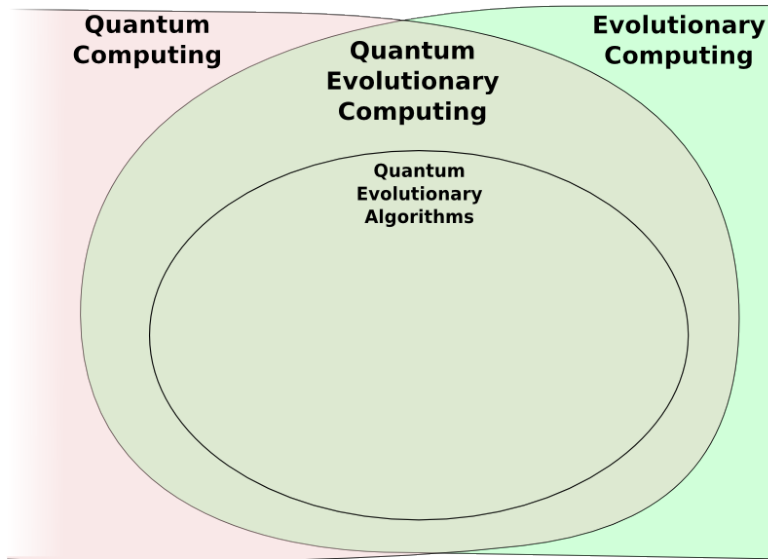
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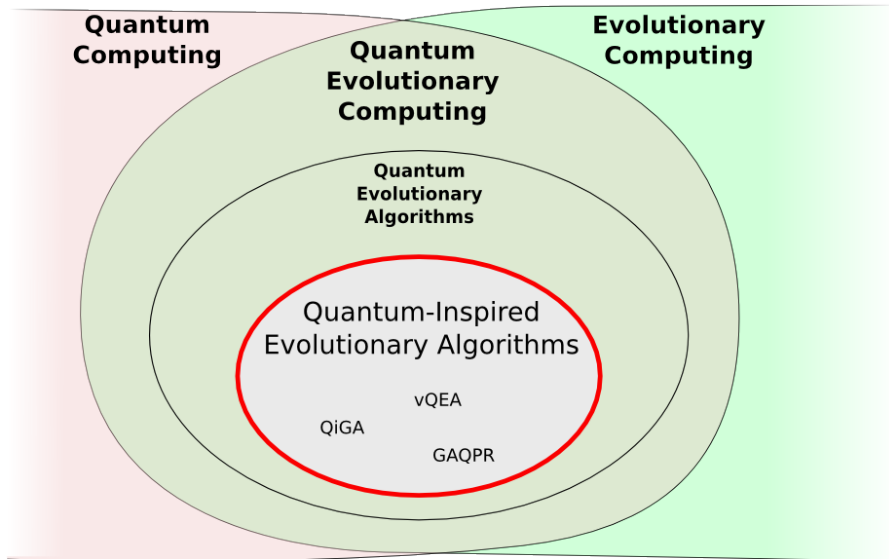
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# LITERATURE REVIEW

- 1 First suggestion: (Narayanan, 1996)
- 2 First early studies: (Han and Kim, 2000)
- 3 Currently, over 160 papers
  - Quantum binary coding: about 130 papers
  - Quantum real coding: 12 papers
  - Applications: 79
  - Theoretical analysis: 5 papers
  - Surveys: 2 papers

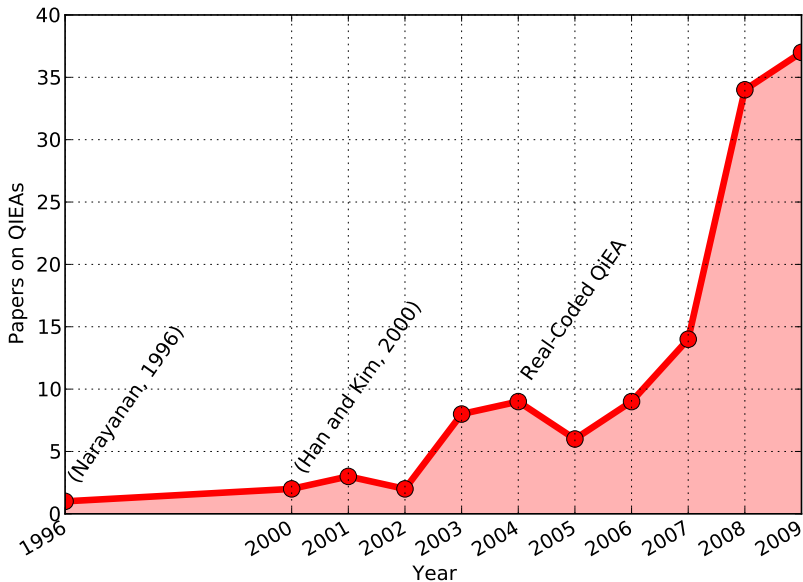
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## ANNUAL DISTRIBUTION OF PAPERS ON QIEAs



## QUANTUM ELEMENTS IN EVOLUTIONARY ALGORITHMS

**1** Representation of solutions

Instead of exact points in a search space,  
*probability distributions* of sampling the space

**2** Initialization**3** Genetic operators**4** Fitness evaluation

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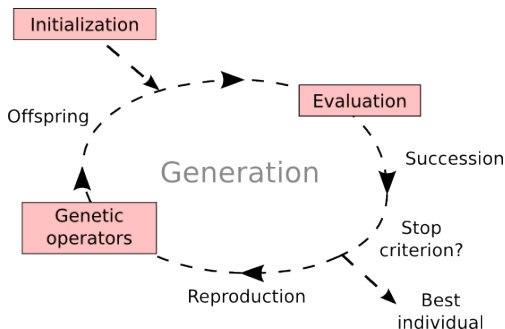
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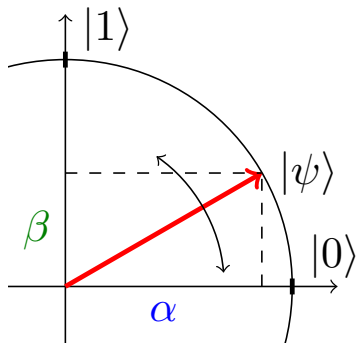


# QUBITS AND BINARY QUANTUM GENES

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- qubit (quantum bit):  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$   
 where:  $\alpha, \beta \in \mathbb{C}$ ,  $|\alpha|^2 + |\beta|^2 = 1$
- $Pr_{|\psi\rangle} : \mathcal{F}_{\{0,1\}} \mapsto [0, 1]$   
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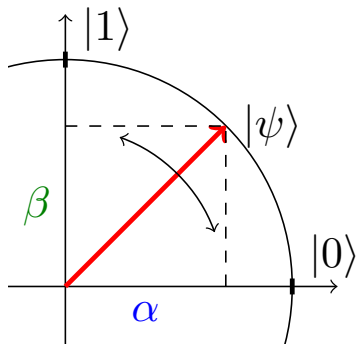
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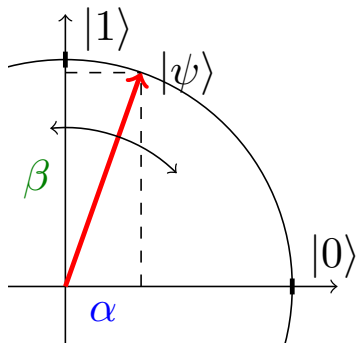
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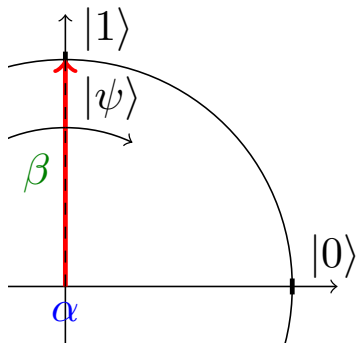
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$$\text{population} \left\{ \begin{array}{l}
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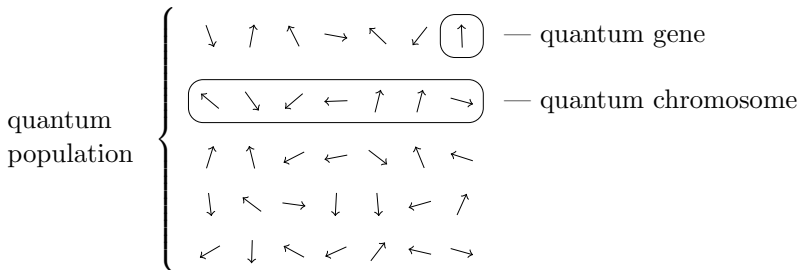
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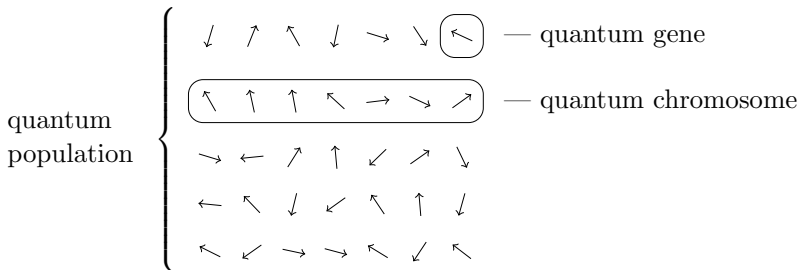
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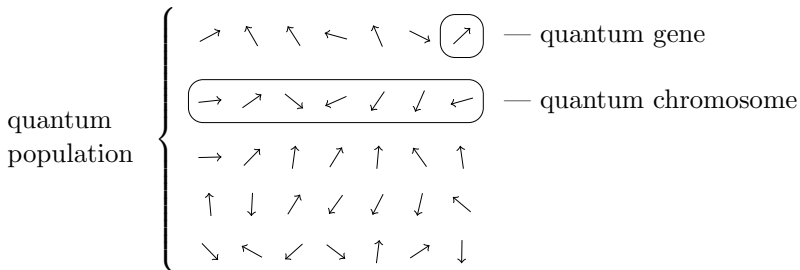
## ILLUSTRATION OF QUANTUM POPULATION



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## PROBLEMS

- 1 How to use the "*new dimension*" efficiently?
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- 1 Convergence analysis of Quantum-Inspired Evolutionary Algorithms based on **Banach's fixed point theorem**
- 2 **Building blocks propagation analysis** in Quantum-Inspired Genetic Algorithms<sup>[1]</sup>
- 3 Tuning QIEAs: **meta-optimization**<sup>[2]</sup>

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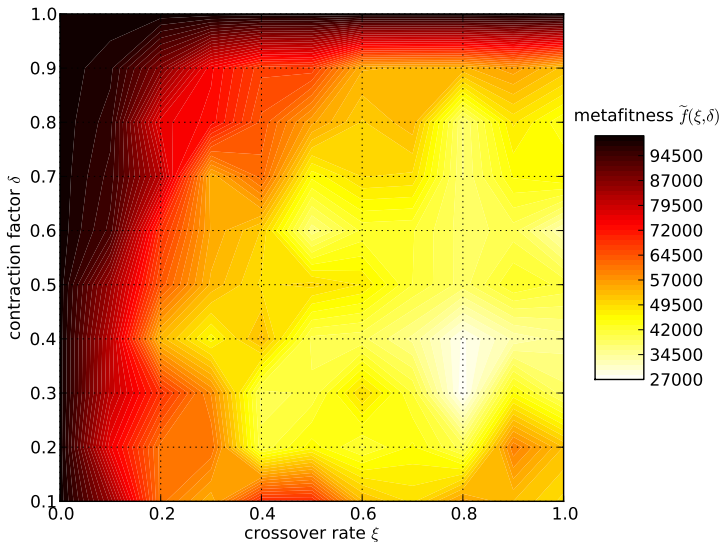
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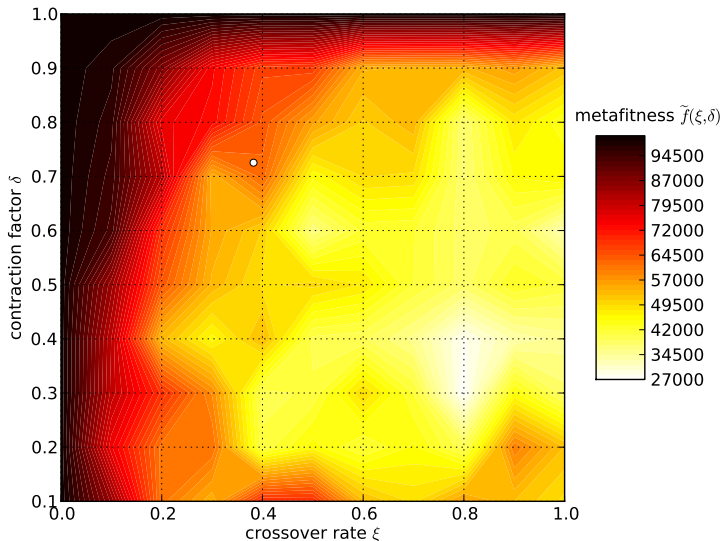
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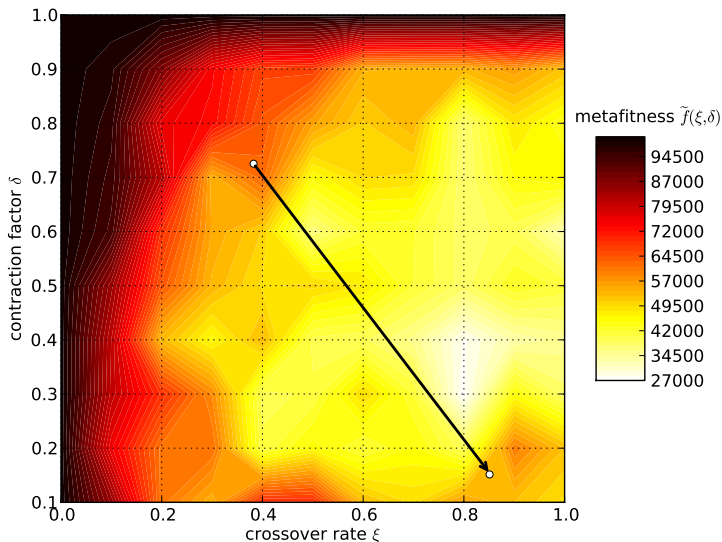


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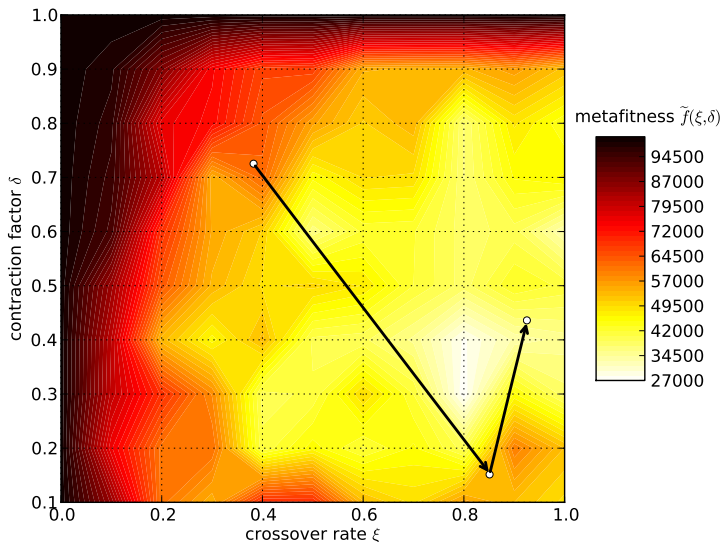
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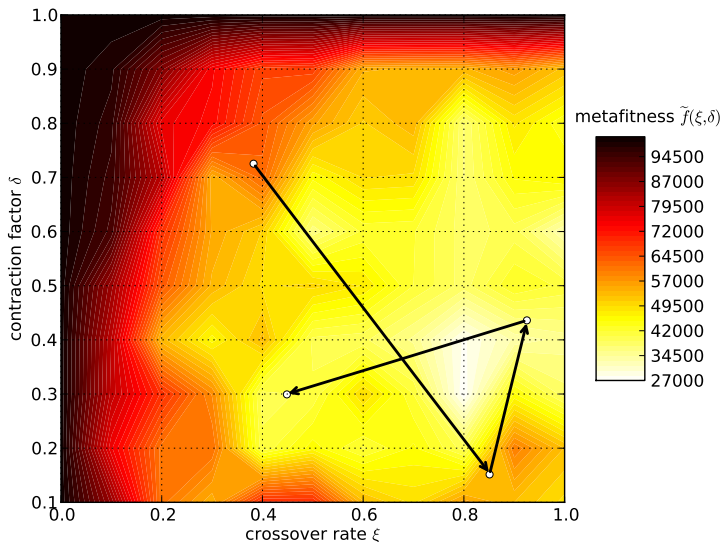


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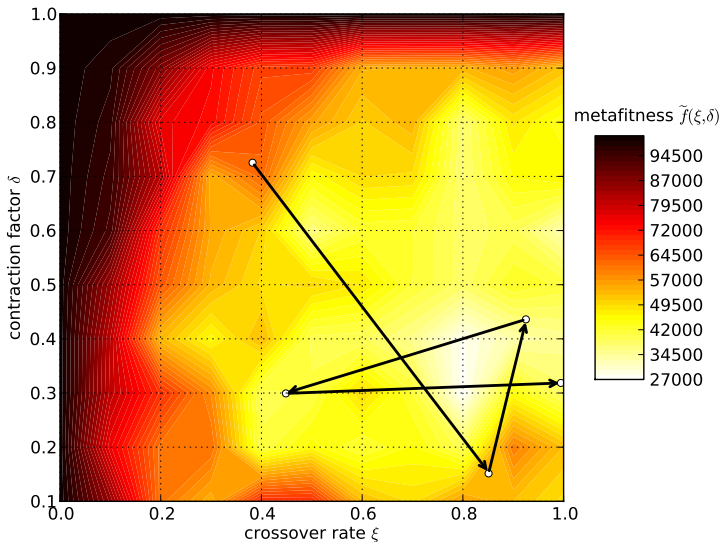
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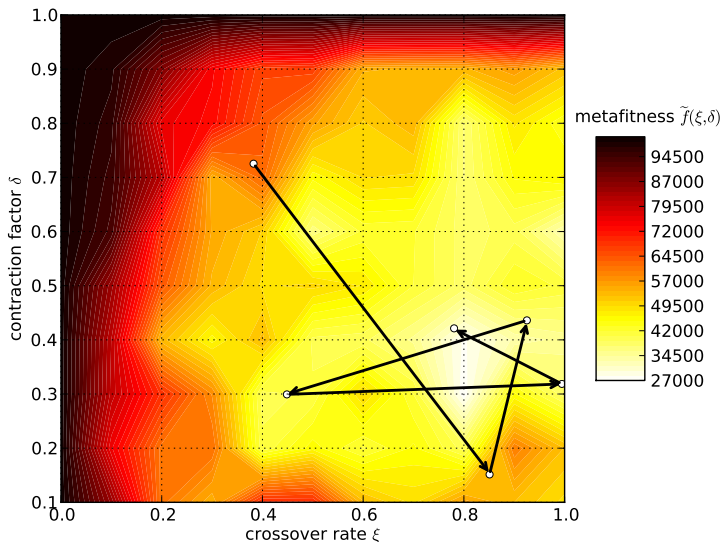
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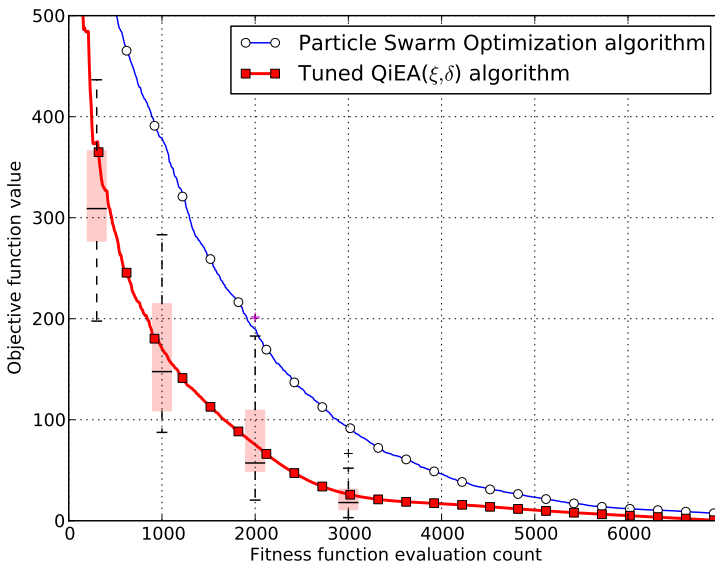
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# BENCHMARKS AND SELECTED APPLICATIONS

- 1 Optimization test suites: "De Jong's suite", CEC '05 suite
- 2 Simultaneous Localization and Mapping (SLAM) problem for mobile robots<sup>[1]</sup>
- 3 Feature selection problem for classifiers performing image segmentation<sup>[2]</sup>
- 4 Inherently vague, imprecise, uncertain problems?  
(evolving rough sets, fuzzy rules etc.)

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