Building Blocks Propagation in Quantum-Inspired Genetic Algorithm

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Situation for Simple Genetic Algorithm

Holland's schema theorem

Short, low order, above average schemata receive exponentially increasing trials in subsequent generations of the classical genetic algorithm

population	1	1	1	1	1	0	0	— binary gene
	1	1	1	1	1	1	0	— chromosome
	0	1	0	1	0	1	0	
	1	1	0	0	0	1	0	
	0	1	1	0	0	0	1	
schema	1	*	0	*	*	*	*	

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schema	1	*	0	*	*	*	*	**1 match**

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	1	1	0	0	0	1	0	
	0	0	1	0	1	1	0	
schema	1	*	0	*	*	*	*	

2 matches

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population:

1 1 0 1 0 1 0 — binary gene
1 0 0 1 0 0 0 — chromosome
0 0 1 0 1 1 0
1 0 0 0 1 0 1
0 0 1 0 0 0 1

schema: 1 * 0 * * * *

3 matches

Qubits and Binary Quantum Genes

- qubit (quantum bit): $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  where: $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$
- $Pr_{|\psi\rangle} : \mathcal{F}_{\{0,1\}} \mapsto [0, 1]$
  $Pr_{|\psi\rangle}(\{0\}) = |\alpha|^2$
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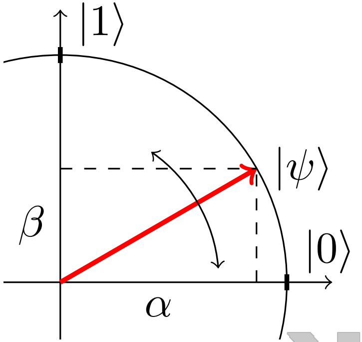
$$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$



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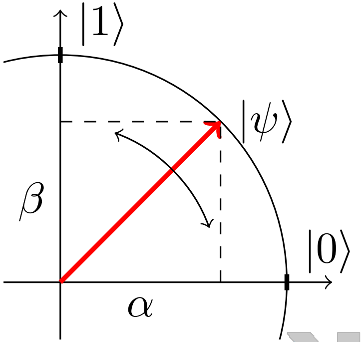

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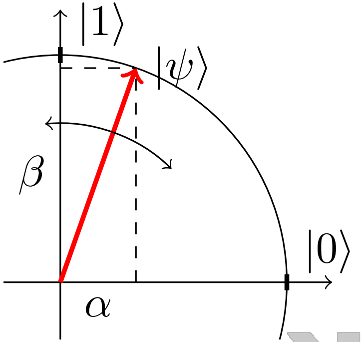
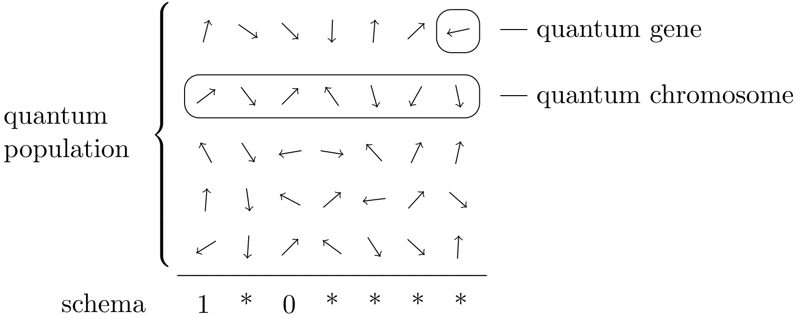
$$|\psi\rangle = \frac{1}{3}|0\rangle + \frac{2\sqrt{2}}{3}|1\rangle$$



Schemata for Quantum Genetic Algorithm



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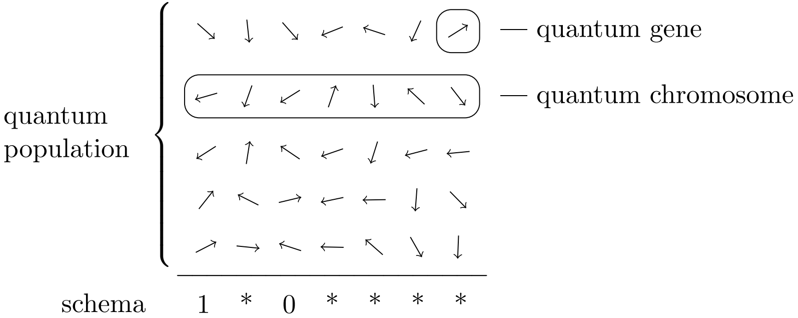

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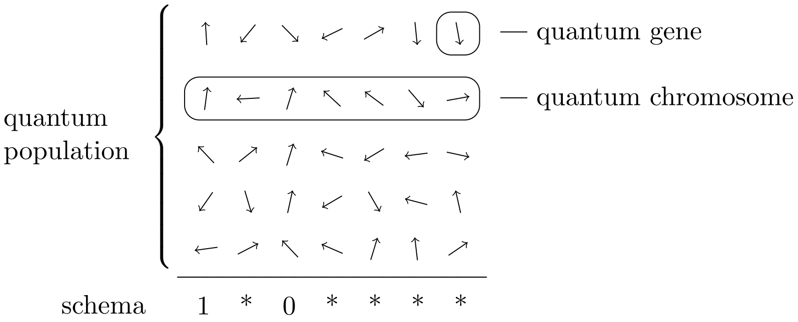

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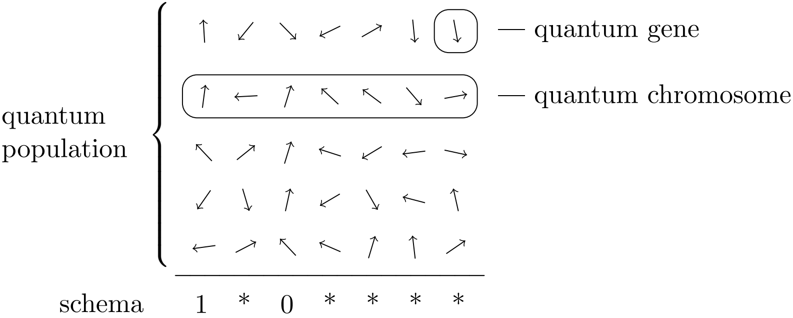


Problem: How many chromosomes match the schema?

Quantum Chromosomes Matching The Schema

Proposal

L – random variable corresponding to the number of binary quantum chromosomes matching the schema

Quantum Chromosomes Matching The Schema

Expected number:

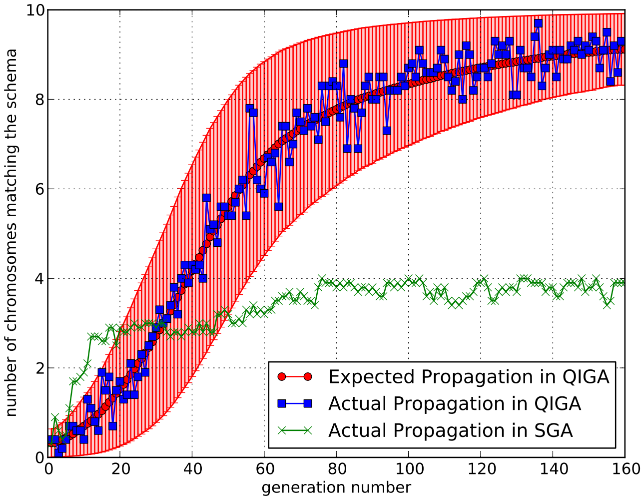
$$E(L) = \sum_{w=0}^{N} \left(w \cdot \sum_{C \in \left\{ X \in 2^{\{1,\ldots,N\}} : |X| = w \right\}} \left(\prod_{j \in C} M(q_i, S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k, S)) \right) \right)$$

Variation:

$$\begin{aligned} V(L) &= \sum_{w=0}^{N} \left(w^2 \sum_{C \in \left\{ X \in 2^{\{1,\ldots,N\}} : |X| = w \right\}} \left(\prod_{j \in C} M(q_j, S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k, S)) \right) \right) \\ &- \left(\sum_{w=0}^{N} \left(w \sum_{C \in \left\{ X \in 2^{\{1,\ldots,N\}} : |X| = w \right\}} \left(\prod_{j \in C} M(q_j, S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k, S)) \right) \right) \right)^2 \end{aligned}$$

where:
N – population size,
$M(q, S) = \prod_{i=1}^{m} Pr_{gi}(\{S[i]\})$ – probability that q matches S
m – length of chromosomes

Building Blocks Propagation Comparison



BB: 01001***************, popsize = 10, chromlen = 20

Thank you for your attention