

# Building Blocks Propagation in Quantum-Inspired Genetic Algorithm

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# Situation for Simple Genetic Algorithm

## Holland's schema theorem

Short, low order, above average schemata receive exponentially increasing trials in subsequent generations of the classical genetic algorithm

population	{	1 1 1 1 1 0 0	— binary gene
		1 1 1 1 1 1 0	— chromosome
		0 1 0 1 0 1 0	
		1 1 0 0 0 1 0	
		0 1 1 0 0 0 1	
schema		1 * 0 * * * *	



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		1 1 1 0 0 1 0	
		1 1 0 0 0 1 0	
		0 0 1 0 1 1 0	
schema		1 * 0 * * * *	<b>2 matches</b>



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		1 0 0 1 0 0 0	— chromosome
		0 0 1 0 1 1 0	
		1 0 0 0 1 0 1	
		0 0 1 0 0 0 1	
schema		1 * 0 * * * *	<b>3 matches</b>



# Qubits and Binary Quantum Genes

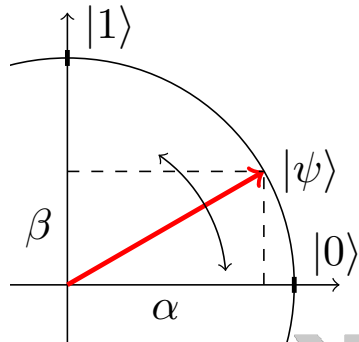
- qubit (quantum bit):  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$   
where:  $\alpha, \beta \in \mathbb{C}$ ,  $|\alpha|^2 + |\beta|^2 = 1$
- $Pr_{|\psi\rangle} : \mathcal{F}_{\{0,1\}} \mapsto [0, 1]$   
 $Pr_{|\psi\rangle}(\{0\}) = |\alpha|^2$   
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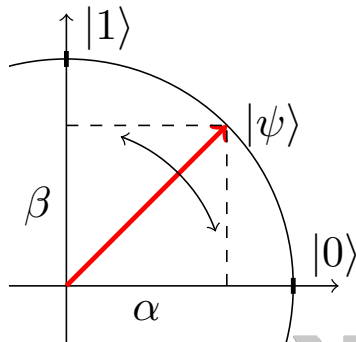
$$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$



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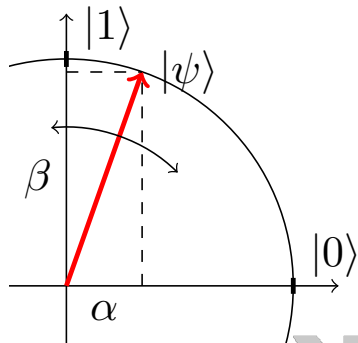
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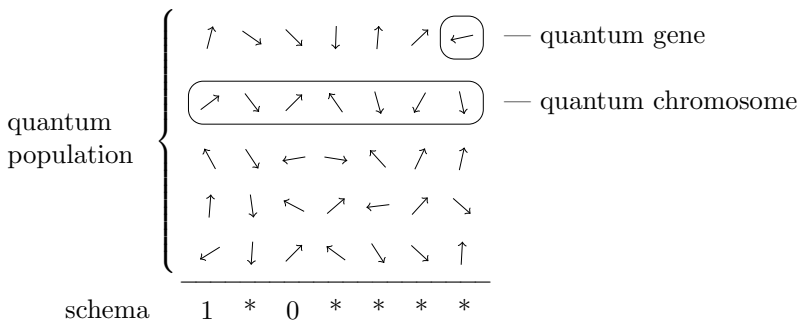
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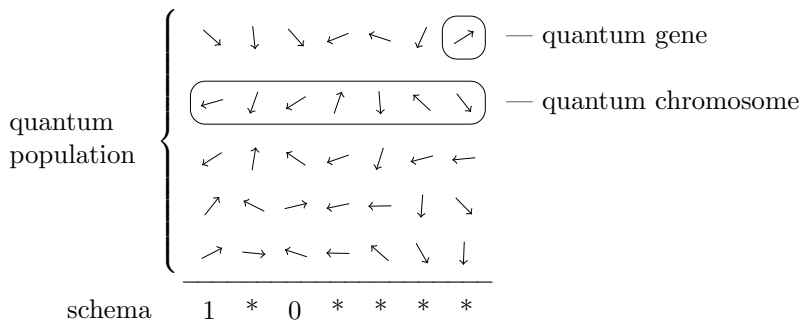
$$|\psi\rangle = \frac{1}{3}|0\rangle + \frac{2\sqrt{2}}{3}|1\rangle$$



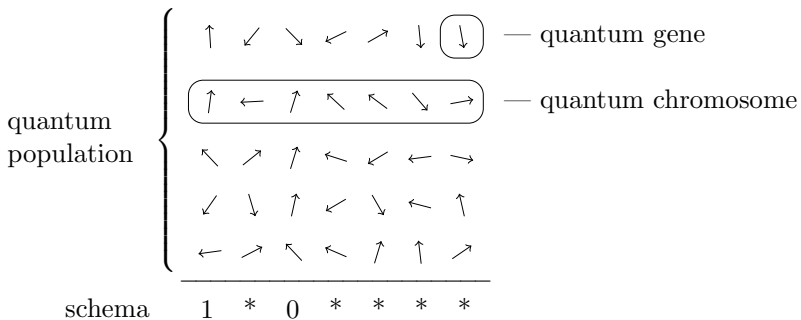
# Schemata for Quantum Genetic Algorithm



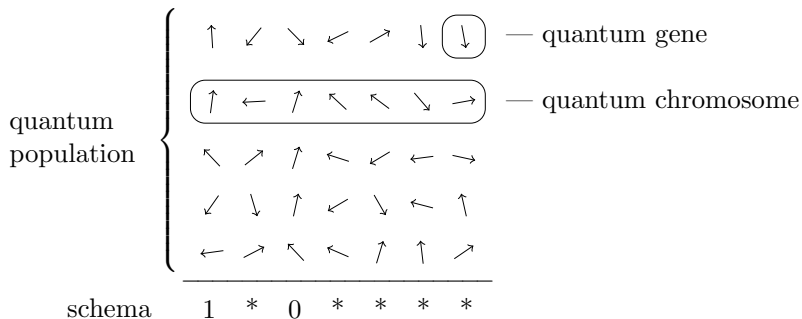
# Schemata for Quantum Genetic Algorithm



# Schemata for Quantum Genetic Algorithm



# Schemata for Quantum Genetic Algorithm



**Problem: How many chromosomes match the schema?**



# Quantum Chromosomes Matching The Schema

## Proposal

- $L$  – random variable corresponding to the number of binary quantum chromosomes matching the schema



# Quantum Chromosomes Matching The Schema

Expected number:

$$E(L) = \sum_{w=0}^N \left( w \cdot \sum_{C \in \{X \in 2^{\{1, \dots, N\}}; |X|=w\}} \left( \prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right)$$

Variation:

$$V(L) = \sum_{w=0}^N \left( w^2 \sum_{C \in \{X \in 2^{\{1, \dots, N\}}; |X|=w\}} \left( \prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right) - \left( \sum_{w=0}^N \left( w \sum_{C \in \{X \in 2^{\{1, \dots, N\}}; |X|=w\}} \left( \prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right) \right)^2$$

where:

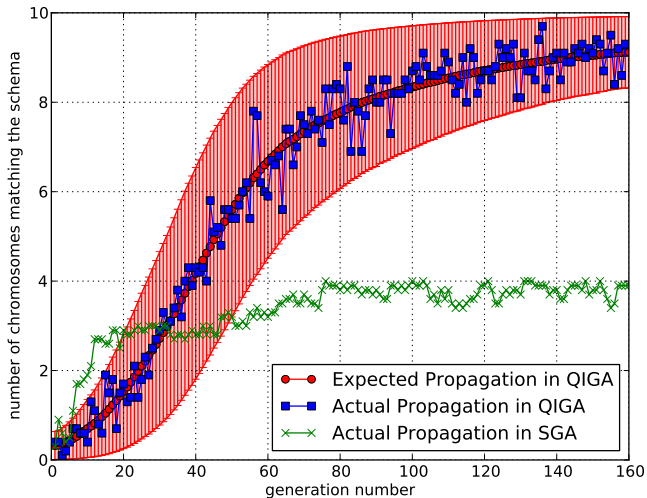
$N$  – population size,

$M(q, S) = \prod_{i=1}^m Pr_{gi}(\{S[i]\})$  – probability that  $q$  matches  $S$

$m$  – length of chromosomes



# Building Blocks Propagation Comparison



BB: 01001\*\*\*\*\* , popsize = 10, chromlen = 20



Thank you for your attention

