

BUILDING BLOCKS PROPAGATION IN QUANTUM-INSPIRED GENETIC ALGORITHM



Robert Nowotniak, Jacek Kucharski

Computer Engineering Department, Technical University of Lodz

{rnowotniak, jkuchars}@kis.p.lodz.pl

1. Quantum-Inspired Genetic Algorithm

In Quantum-Inspired Genetic Algorithm (QIGA) a novel representation of solutions is employed. Genes are modelled upon the concept of qubits, which brings an additional element of randomness and a "new dimension" into the algorithm.

The *qubit* is a basic unit of quantum information. It is a normalised vector in a two dimensional vector space spanned by base vectors $|0\rangle$ and $|1\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

where $\alpha, \beta \in \mathbb{C}$, $|0\rangle = [1 \ 0]^T$, $|1\rangle = [0 \ 1]^T$ and $|\alpha|^2 + |\beta|^2 = 1$.

The mapping between binary quantum gene $|\psi\rangle$ and its probability distribution is given as follows: $Pr_{|\psi\rangle}(\{0\}) = |\alpha|^2$, $Pr_{|\psi\rangle}(\{1\}) = |\beta|^2$. Obviously, $Pr(\{0, 1\}) = 1$.

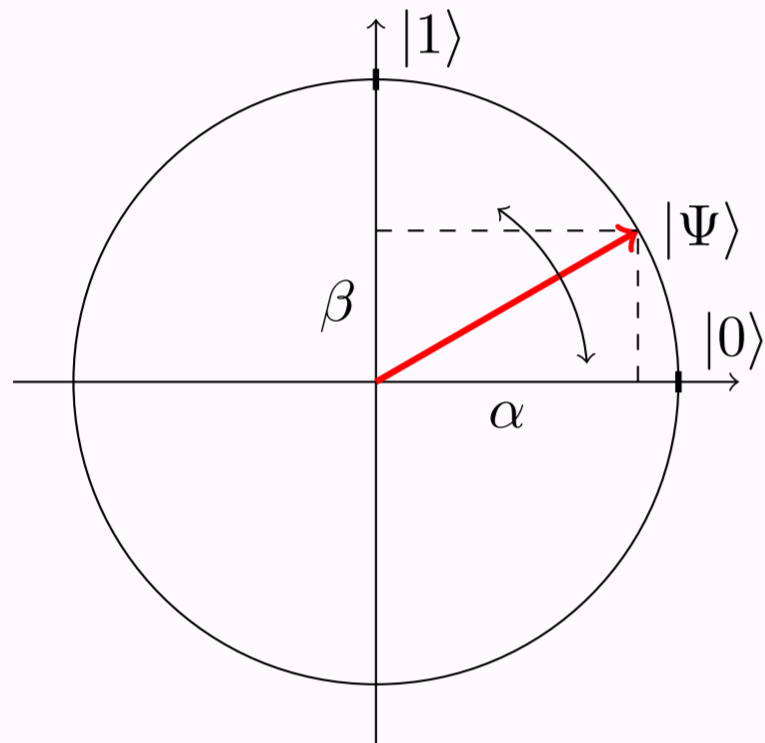


Figure 1: Geometric illustration of binary quantum gene, represented as qubit with its imaginary part omitted. Along with increase of the angle between the vector $|\Psi\rangle$ and the horizontal axis, the probability of observing value 1 grows.

The algorithm uses *binary quantum chromosomes* for representation of solutions, encoded as:

$$q_t = \begin{bmatrix} \alpha_1^t & \alpha_2^t & \dots & \alpha_m^t \\ \beta_1^t & \beta_2^t & \dots & \beta_m^t \end{bmatrix} \quad (2)$$

Thus, a state of quantum population can be depicted by a matrix of arrows as in figure 2.

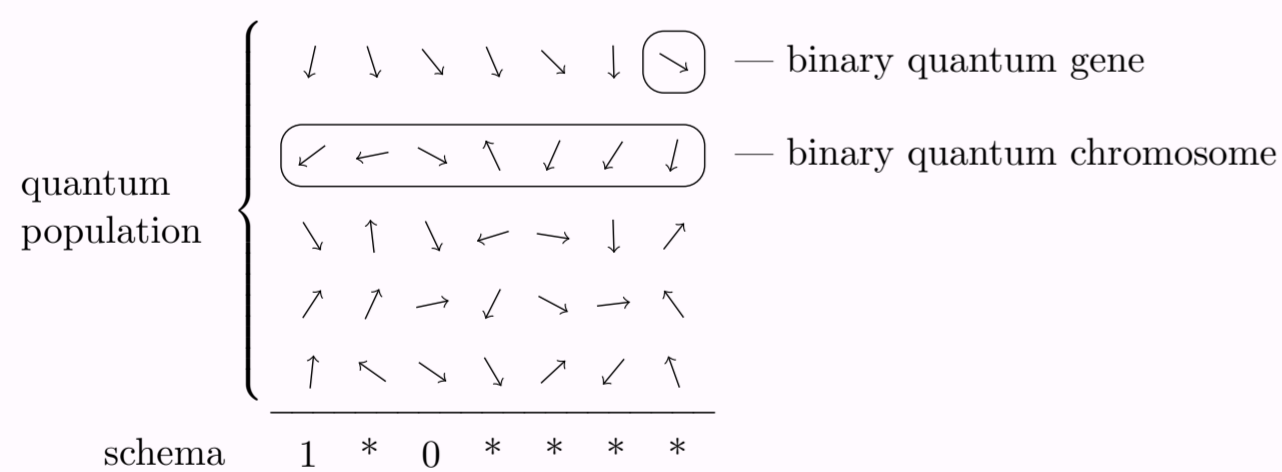


Figure 2: Illustration of quantum population.

The pseudocode of QIGA is given below:

Algorithm 1: Quantum-Inspired Genetic Algorithm

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t ← 0
initialize Q(0)
make P(0) by observing Q(0)
evaluate P(0)
store the best solution among P(0)
while not termination-criterion do
  t ← t + 1
  make P(t) by observing Q(t - 1) states
  evaluate P(t)
  update Q(t) using quantum gates U(t)
  store the best solution among P(t)
end while

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In recent years several extensions of the algorithm have been proposed, including GAQPR, vQEA and NQEA.

2. Schemata and Binary Quantum Chromosomes

The probability $M(q, S)$ that the binary quantum chromosome q matches the schema S of m elements is given by the formula:

$$M(q, S) = \prod_{i=1}^m Pr_{q_i}(\{S[i]\}) \quad (3)$$

where: $S[i]$ – element of the S schema at the position i .

	1	0	$\frac{1}{2} 0\rangle + \frac{\sqrt{3}}{2} 1\rangle$	$M(q, S)$
S_1	*	0	*	1
S_2	1	0	*	1
S_3	*	*	*	1
S_4	*	*	0	$\frac{1}{4}$
S_5	1	*	1	$\frac{3}{4}$

Figure 3: Example for binary quantum chromosome $q = g_1g_2g_3 = 10(\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle)$ and for five different schemata. The probability $M(q, S_1)$ that q matches $S_1 = *0*$ equals 1. The probability $M(q, S_5)$ that q matches $S_5 = 1*1$ equals $\frac{3}{4}$ etc.

The expected number $E(L)$ of quantum chromosomes matching the schema S in the quantum population $Q = \{q_1, \dots, q_N\}$ is expressed as:

$$E(L) = \sum_{w=0}^N \left(w \cdot \sum_{C \in \{X \in \{2^{1, \dots, N}\} : |X|=w\}} \left(\prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right)$$

The variance of the random variable:

$$V(L) = \sum_{w=0}^N \left(w^2 \sum_{C \in \{X \in \{2^{1, \dots, N}\} : |X|=w\}} \left(\prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right) - \left(\sum_{w=0}^N \left(w \sum_{C \in \{X \in \{2^{1, \dots, N}\} : |X|=w\}} \left(\prod_{j \in C} M(q_j, S) \prod_{k \in \{1, \dots, N\} \setminus C} (1 - M(q_k, S)) \right) \right) \right)^2$$

3. Test Problem

To analyse propagation of building blocks, a numerical experiment has been conducted for simple optimization test problem. The landscape of the test function, created by B-spline interpolation, has been presented in figure 4.

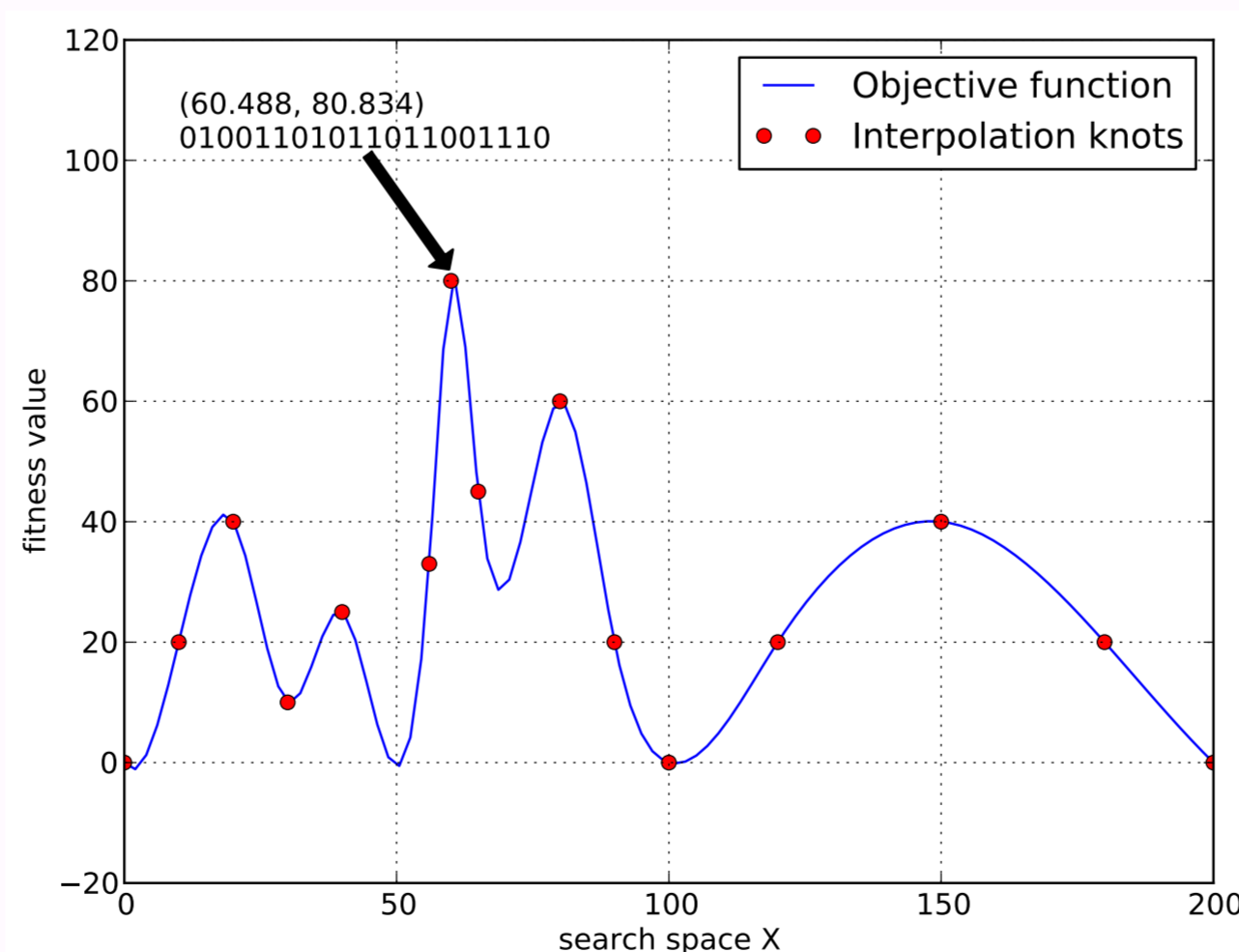


Figure 4: Landscape of the test fitness function

The global maximum of the test function is at $x^* = 60.488$, where $f(x^*) = 80.834$. Natural binary coding has been applied and the binary string which encodes the best solution is 01001101011011001110. The short, low order, above average schema selected as a building block is 01001*****.

4. Results of The Experiment

In the experiment performance of QIGA has been compared with Simple Genetic Algorithm for the test problem. In figure 5 comparison plot has been presented. The plot is an average over 10 runs of the algorithms.

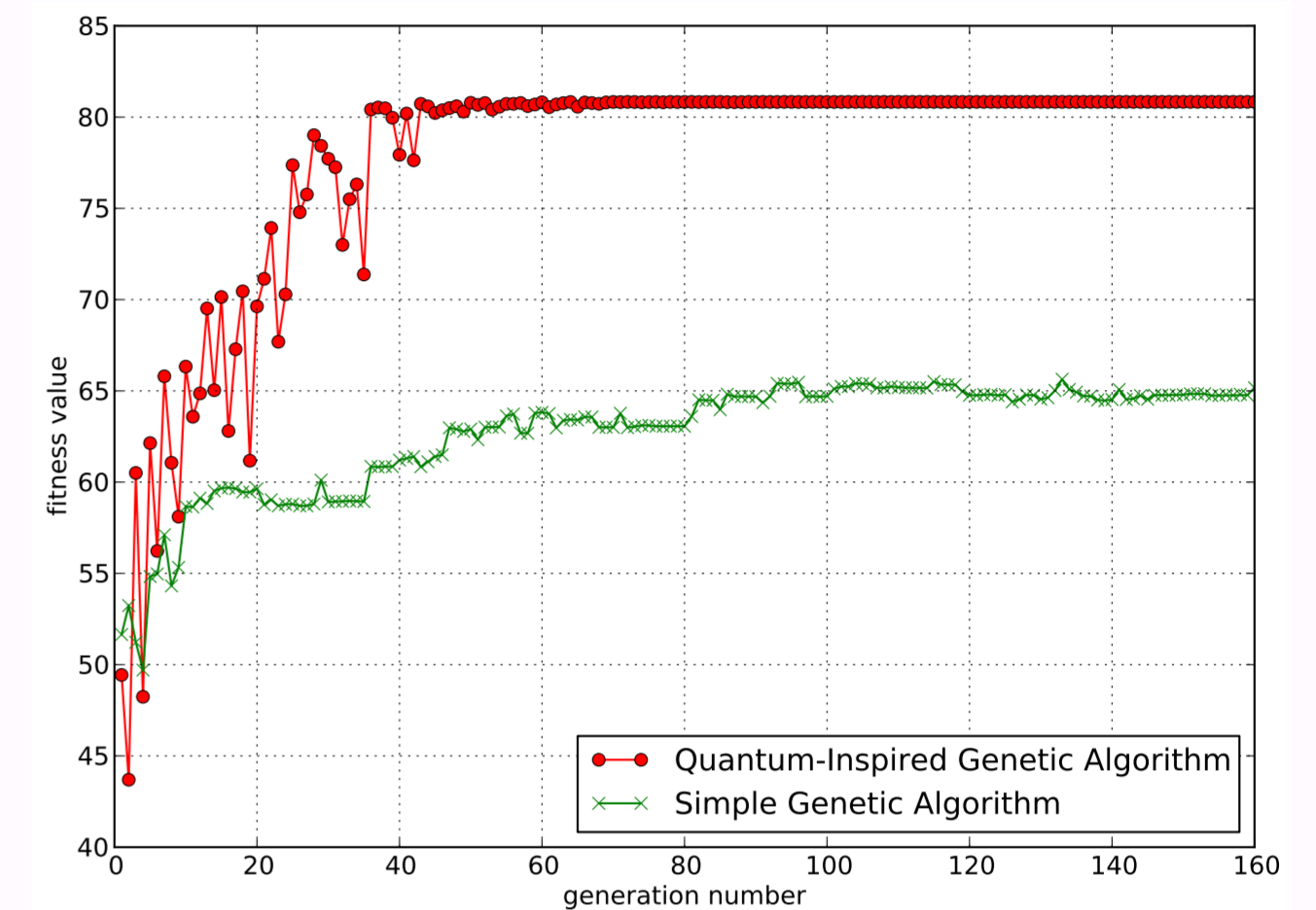


Figure 5: Efficiency comparison of the two algorithms, fitness value per generation number

It is easy to notice that QIGA outperforms SGA on the test problem easily. In about 40 generation QIGA always found the optimum solution.

In figure 6 expected and actual propagation of the selected building block in QIGA and SGA have been presented. Much better propagation has been observed in QIGA.

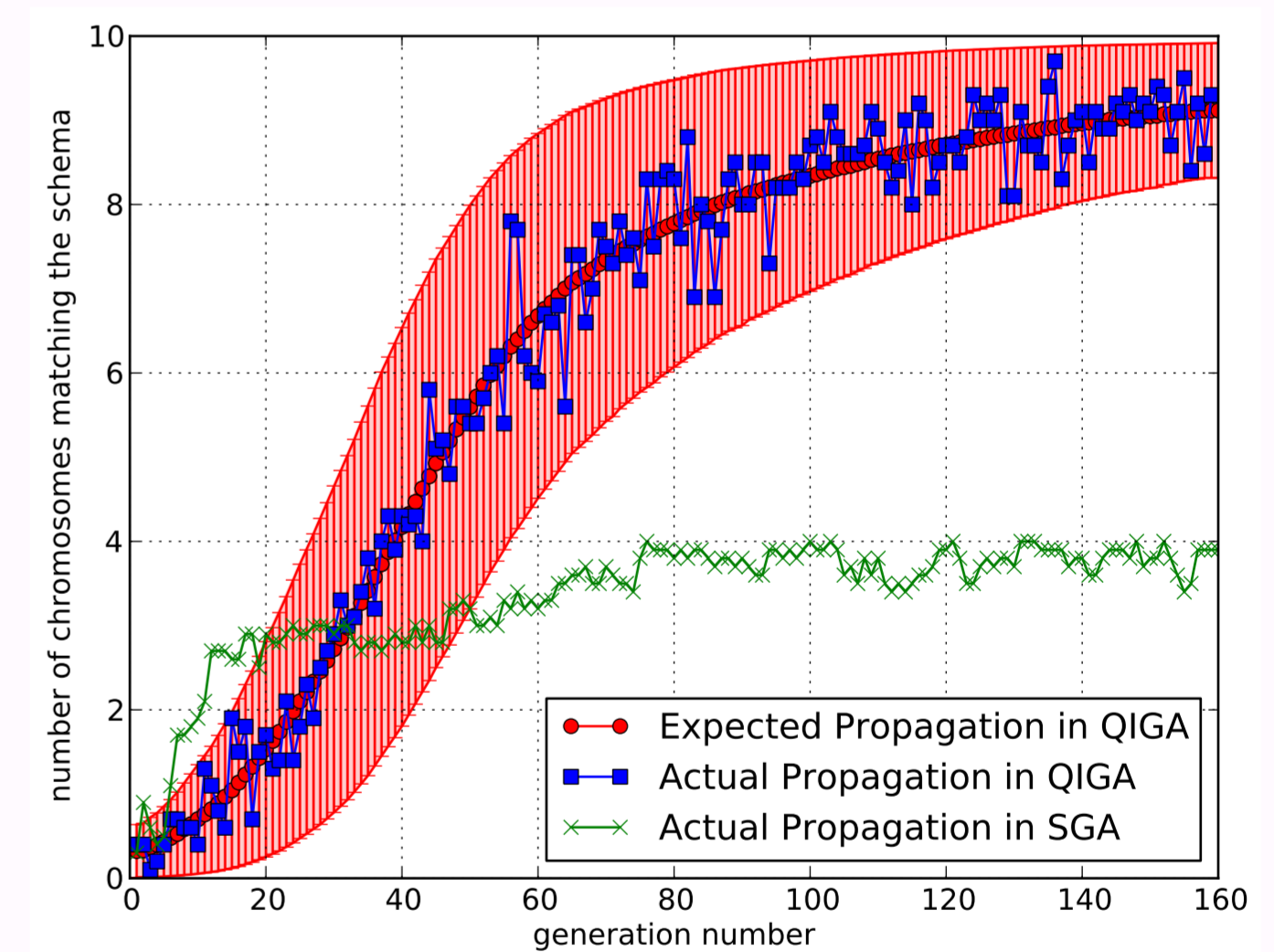


Figure 6: Number of quantum chromosomes matching the schema per generation number. Comparison of propagation in QIGA and SGA.

The actual propagation in QIGA verifies that the formulas, which express expected propagation, have been devised correctly. Figure 7 presents how probability distribution of selected binary quantum chromosome "covers" a domain of the test function.

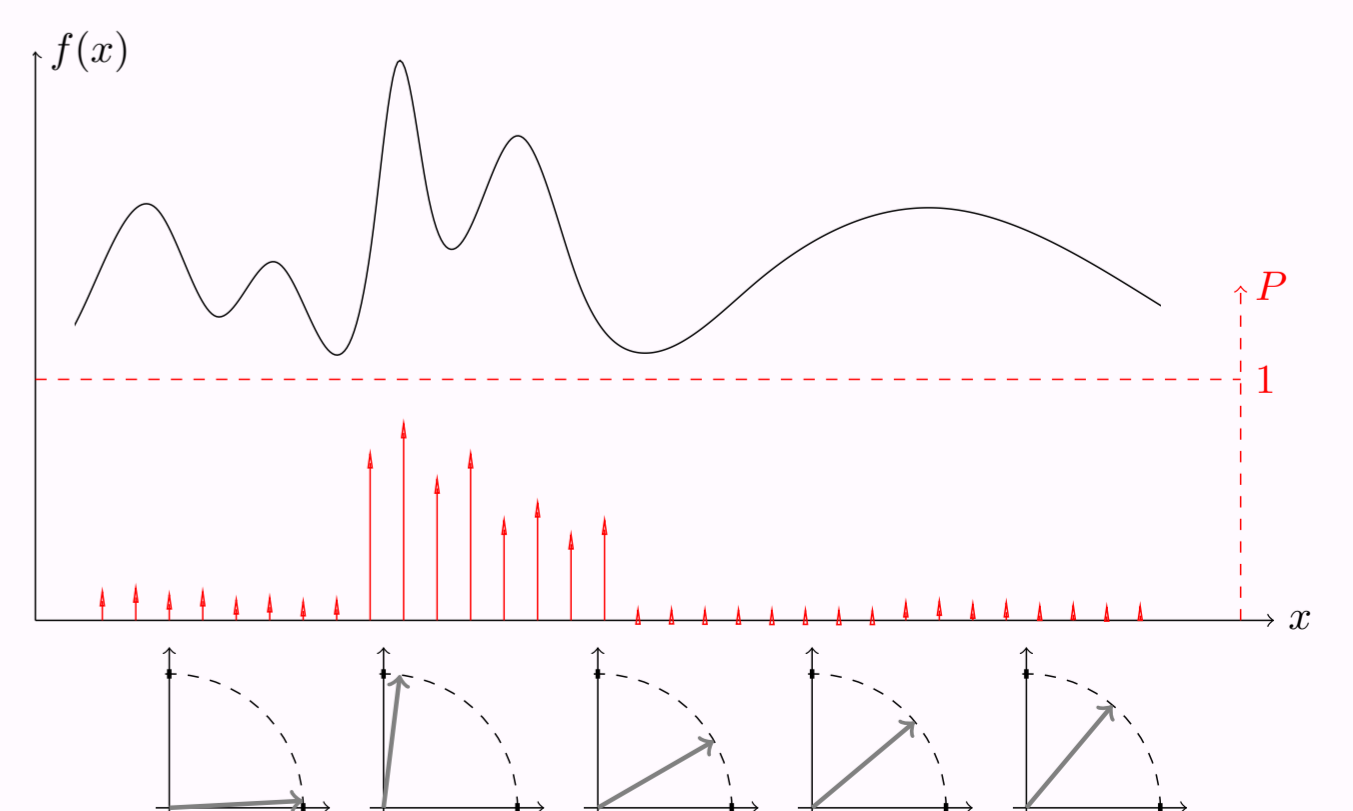


Figure 7: Illustration of selected quantum chromosome of five genes and corresponding probability distribution of sampling the search space