Building Blocks Propagation in Quantum-Inspired Genetic Algorithm

Robert Nowotniak, Jacek Kucharski

Computer Engineering Department, Technical University of Lodz

{rnowotniak,jkuchars}@kis.p.lodz.pl

1. Quantum-Inspired Genetic Algorithm

In Quantum-Inspired Genetic Algorithm (QIGA) a novel representation of solutions is employed. Genes are modelled upon the concept of qubits, which brings an additional element of randomness and a "new dimension" into the algorithm.

The *qubit* is a basic unit of quantum information. It is a normalised vector in a two dimensional vector space spanned by base vectors $|0\rangle$ and $|1\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

where $\alpha, \beta \in \mathbb{C}$, $|0\rangle = [1 \quad 0]^T$, $|1\rangle = [0 \quad 1]^T$ and $|\alpha|^2 + |\beta|^2 = 1$.

The mapping between binary quantum gene $|\psi\rangle$ and its probability distribution is given as follows: $Pr_{|\psi\rangle}(\{0\}) = |\alpha|^2$, $Pr_{|\psi\rangle}(\{1\}) = |\beta|^2$. Obviously, $Pr(\{0,1\}) = 1$.

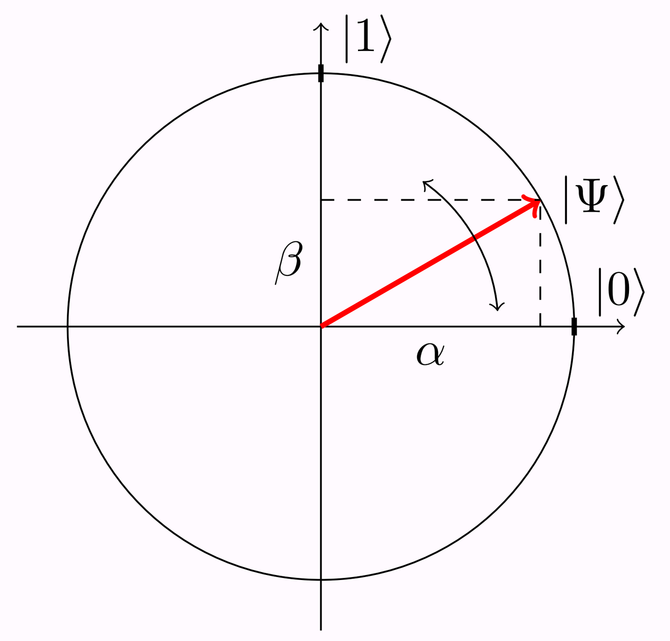

Figure 1: *Geometric illustration of binary quantum gene, represented as qubit with its imaginary part omitted. Along with increase of the angle between the vector $|\Psi\rangle$ and the horizontal axis, the probability of observing value 1 grows.*

The algorithm uses *binary quantum chromosomes* for representation of solutions, encoded as:

$$q_t = \left[\begin{array}{c|c|c|c} \alpha_1^t & \alpha_2^t & \ldots & \alpha_m^t \\ \beta_1^t & \beta_2^t & \ldots & \beta_m^t \end{array} \right] \tag{2}$$

Thus, a state of quantum population can be depicted by a matrix of arrows as in figure 2.

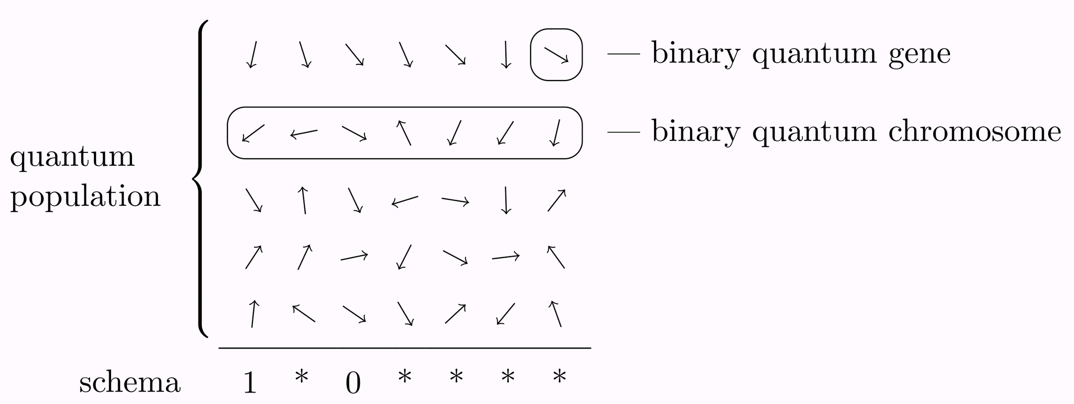

Figure 2: *Illustration of quantum population.*

The pseudocode of QIGA is given below:

Algorithm 1: *Quantum-Inspired Genetic Algorithm*

$t \leftarrow 0$
initialize $Q(0)$
make $P(0)$ by observing $Q(0)$
evaluate $P(0)$
store the best solution among $P(0)$
while not termination-criterion **do**
    $t \leftarrow t+1$
    make $P(t)$ by observing $Q(t-1)$ states
    evaluate $P(t)$
    update $Q(t)$ using quantum gates $U(t)$
    store the best solution among $P(t)$
end while

In recent years several extensions of the algorithm have been proposed, including GAQPR, vQEA and NQEA.

2. Schemata and Binary Quantum Chromosomes

The probability $M(q,S)$ that the binary quantum chromosome q matches the schema S of m elements is given by the formula:

$$M(q,S) = \prod_{i=1}^{m} Pr_{gi}\left(\{S[i]\}\right) \tag{3}$$

where: $S[i]$ – element of the S schema at the position i.

	1	0	$\frac{1}{2}\vert 0\rangle + \frac{\sqrt{3}}{2}\vert 1\rangle$	M(q,S)
S_1	*	0	*	1
S_2	1	0	*	1
S_3	*	*	*	1
S_4	*	*	0	$\frac{1}{4}$
S_5	1	*	1	$\frac{3}{4}$

Figure 3: *Example for binary quantum chromosome $q = g_1 g_2 g_3 = 10(\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle)$ and for five different schemata. The probability $M(q,S_1)$ that q matches $S_1 = *0*$ equals 1. The probability $M(q,S_5)$ that q matches $S_5 = 1*1$ equals $\frac{3}{4}$ etc.*

The expected number $E(L)$ of quantum chromosomes matching the schema S in the quantum population $Q = \{q_1, \ldots, q_N\}$ is expressed as:

$$E(L) = \sum_{w=0}^{N} \left(w \cdot \sum_{C \in \{X \in 2^{\{1,\ldots,N\}} : |X| = w\}} \left(\prod_{j \in C} M(q_i,S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k,S)) \right) \right)$$

The variance of the random variable:

$$V(L) = \sum_{w=0}^{N} \left(w^2 \sum_{C \in \{X \in 2^{\{1,\ldots,N\}} : |X| = w\}} \left(\prod_{j \in C} M(q_j,S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k,S)) \right) \right) - \left(\sum_{w=0}^{N} \left(w \sum_{C \in \{X \in 2^{\{1,\ldots,N\}} : |X| = w\}} \left(\prod_{j \in C} M(q_j,S) \prod_{k \in \{1,\ldots,N\} \setminus C} (1 - M(q_k,S)) \right) \right) \right)^2$$

3. Test Problem

To analyse propagation of building blocks, a numerical experiment has been conducted for simple optimization test problem. The landscape of the test function, created by B-spline interpolation, has been presented in figure 4.

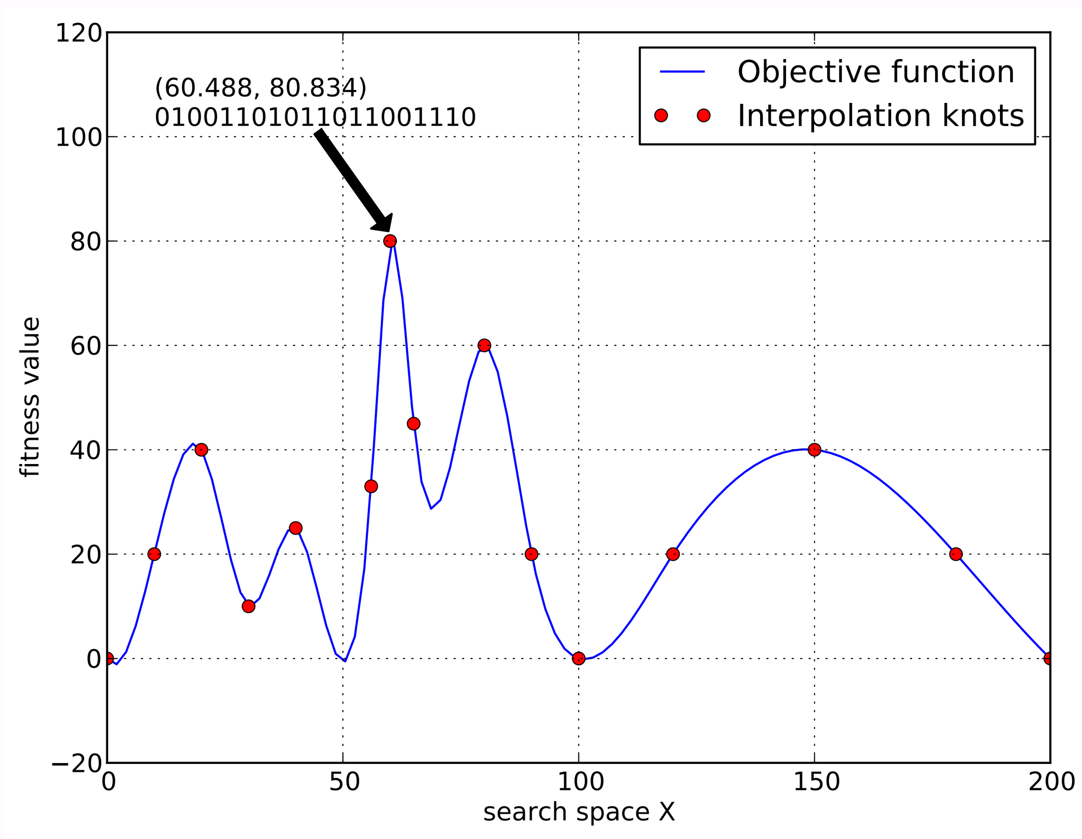

Figure 4: *Landscape of the test fitness function*

The global maximum of the test function is at $x^* = 60.488$, where $f(x^*) = 80.834$. Natural binary coding has been applied and the binary string which encodes the best solution is 01001101011011001110. The short, low order, above average schema selected as a building block is 01001***************.

4. Results of The Experiment

In the experiment performance of QIGA has been compared with Simple Genetic Algorithm for the test problem. In figure 5 comparison plot has been presented. The plot is an average over 10 runs of the algorithms.

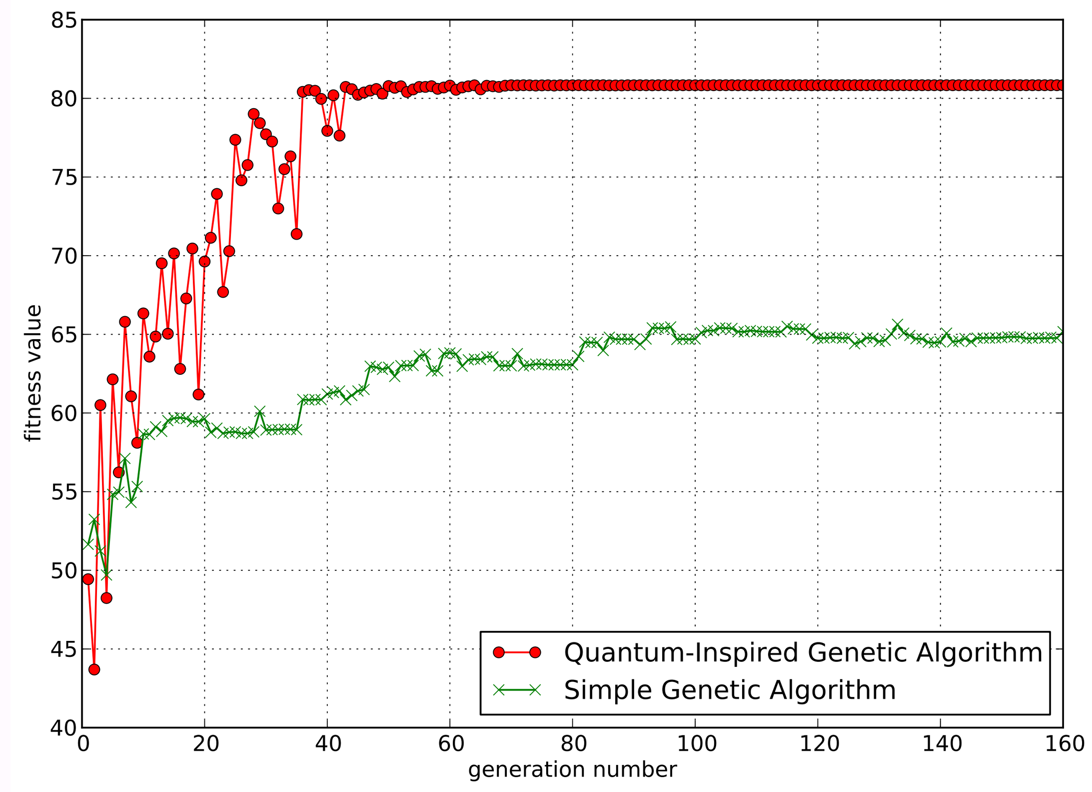

Figure 5: *Efficiency comparison of the two algorithms, fitness value per generation number*

It is easy to notice that QIGA outperforms SGA on the test problem easily. In about 40 generation QIGA always found the optimum solution.

In figure 6 expected and actual propagation of the selected building block in QIGA and SGA have been presented. Much better propagation has been observed in QIGA.

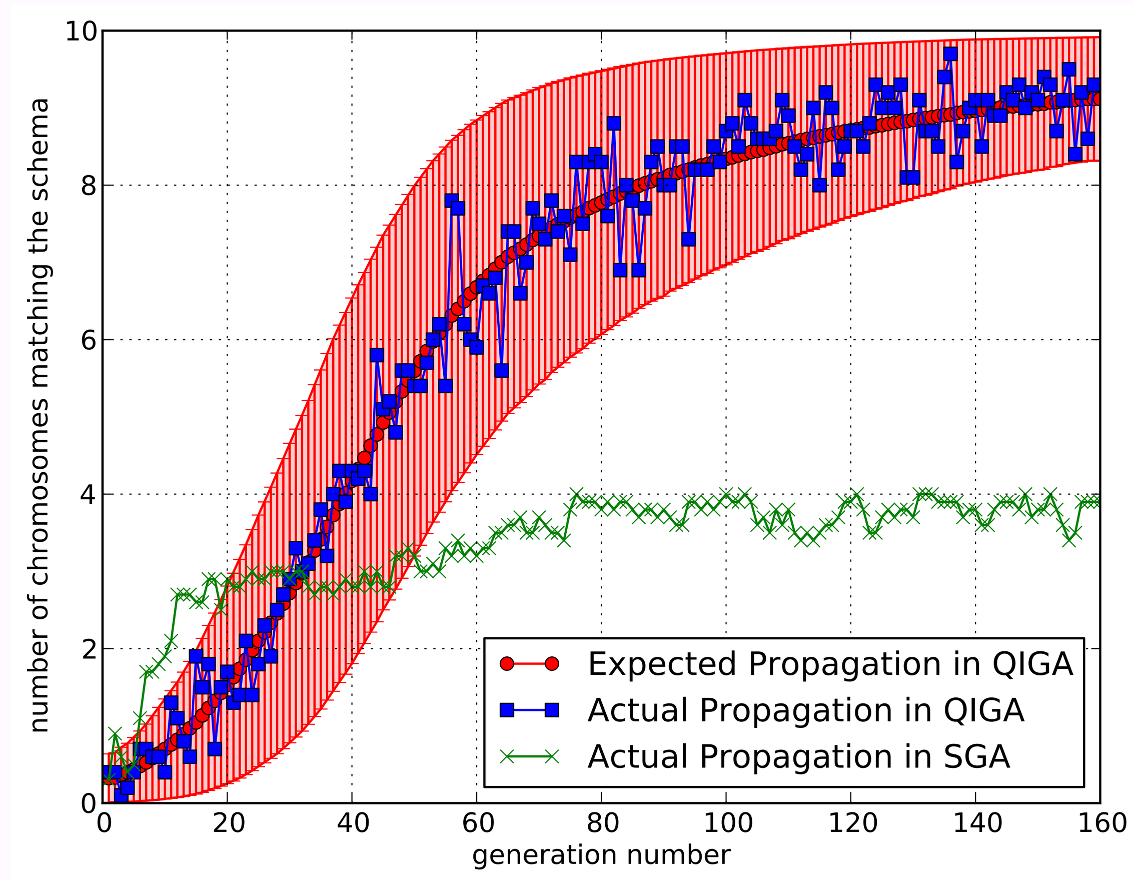

Figure 6: *Number of quantum chromosomes matching the schema per generation number. Comparison of propagation in QIGA and SGA.*

The actual propagation in QIGA verifies that the formulas, which express expected propagation, have been devised correctly. Figure 7 presents how probability distribution of selected binary quantum chromosome "covers" a domain of the test function.

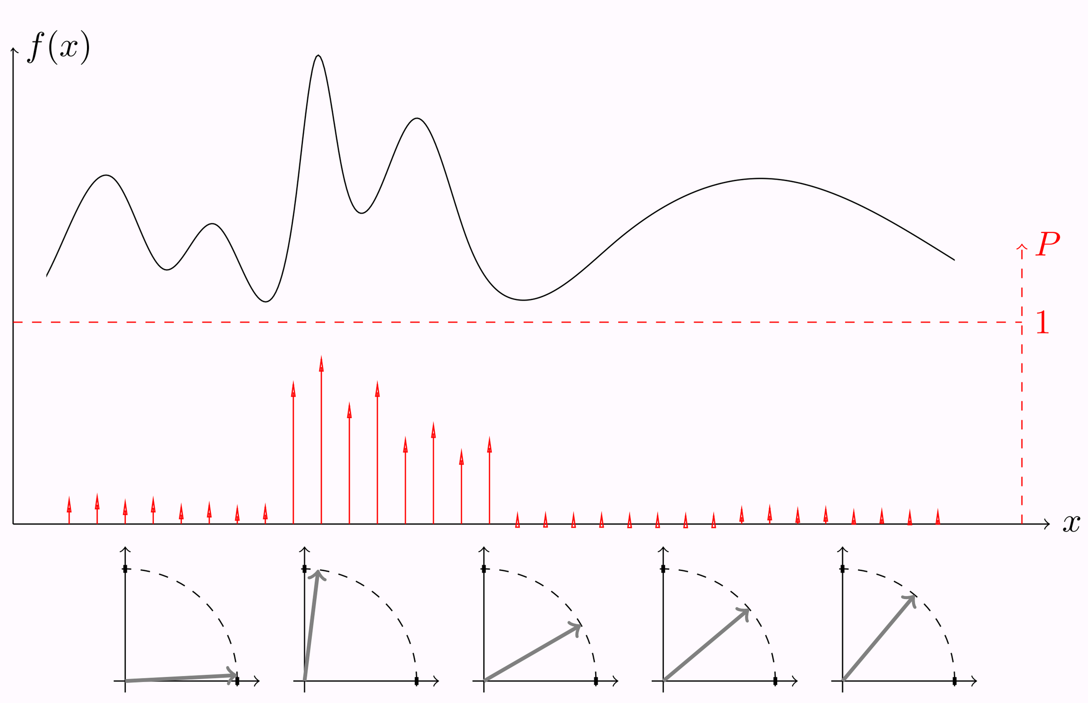

Figure 7: *Illustration of selected quantum chromosome of five genes and corresponding probability distribution of sampling the search space*

SŁOK 2010, Przetwarzanie i analiza sygnałów w systemach wizji i sterowania, 23-25 czerwca 2010, Słok